

Area Properties

Introduction

This learning tool guides you through a two-step process.

- You can define an area on the x - y plane by using an easy-to-learn user interface. The areas can be defined by combining simple geometric shapes (polygon or ellipse) or by specifying the equations ($y=f(x)$, $x=g(y)$) of the bounding curves.
- You can calculate various area properties, such as: area, centroid, and products and moments of inertia. This tool can also demonstrate the use of the Parallel Axis and Rotation Axis Theorems.

Workspace

When you open the tool, you will get the window of Figure-1.

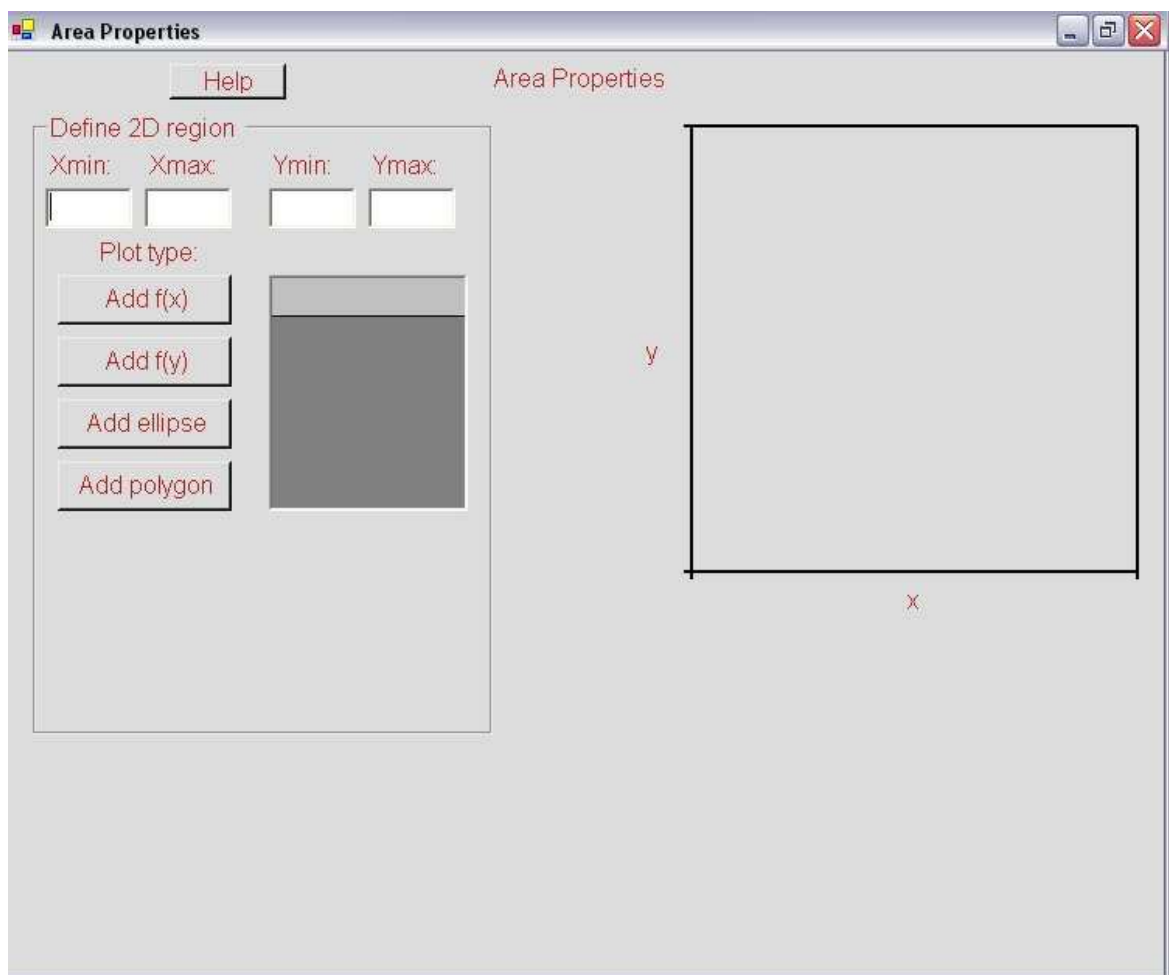


Figure-1: Workspace

Example-1

We wish to determine the properties of the area shown in Figure-2.

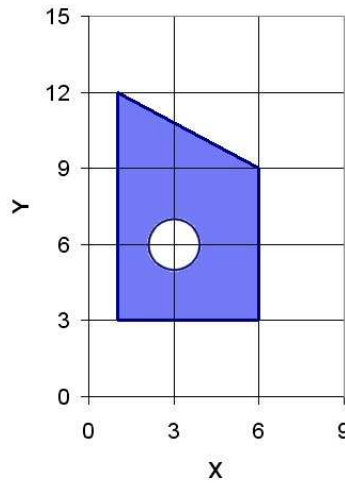


Figure-2: Problem

Enclose the area **in a square of smallest size.**

The span of the area is 5 along x-axis ($1 < x < 6$) and 9 along y-axis ($3 < y < 12$).

Therefore, the size of the chosen square is $\max\{5 \text{ and } 9\} = 9$.

The span of the square is chosen as $1 < x < 10$ and $3 < y < 12$.

Enter $x_{\min}=1$, $x_{\max}=10$, $y_{\min}=3$, and $y_{\max}=12$.

The area consists of a polygon and a circle.

Circle Definition

The circle is of radius 1 with center at (3,6).

Click “Add Ellipse” in the window of Figure-1.

Enter “Ellipse definition”, shown in Figure-3, as - “Center x” = 3, “Center y” = 6, “Semi-major Axis” = 1, “Semi-minor Axis” = 1.

Click “Done”.

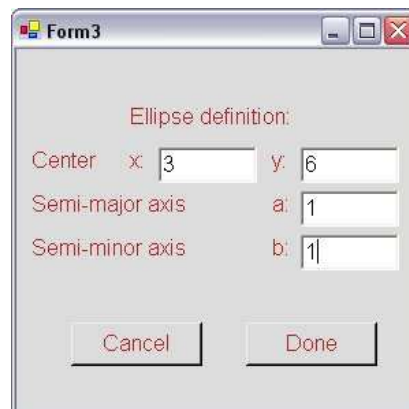


Figure-3: Ellipse definition.

Polygon Definition

Arrange the coordinates of the vertices of the polygon in a **clockwise** manner, starting at any vertex – (1,3), (6,3), (6,9), and (1,12).

Click “Add Polygon” in the window of Figure-1.

Enter “polygon definition”, shown in Figure-4, as: $\text{poly}(4, \{1,6,6,1\}, \{3,3,9,12\})$.

Note: The number 4 identifies the polygon as 4-sided.

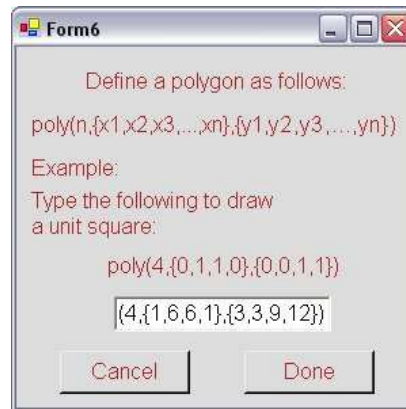


Figure-4: Polygon definition

Click “Done” – You will get the window of Figure-5.

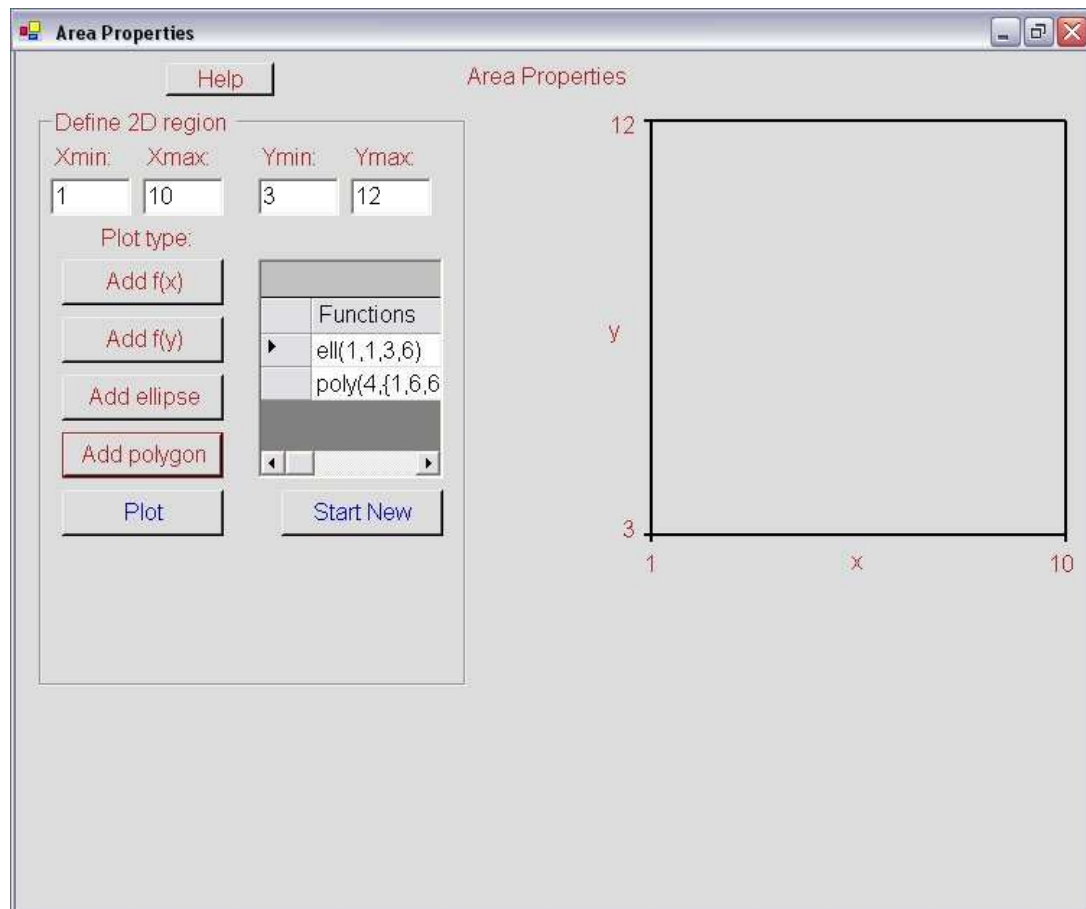


Figure-5: Area data.

Click “Plot” – You will get the window of Figure-6.

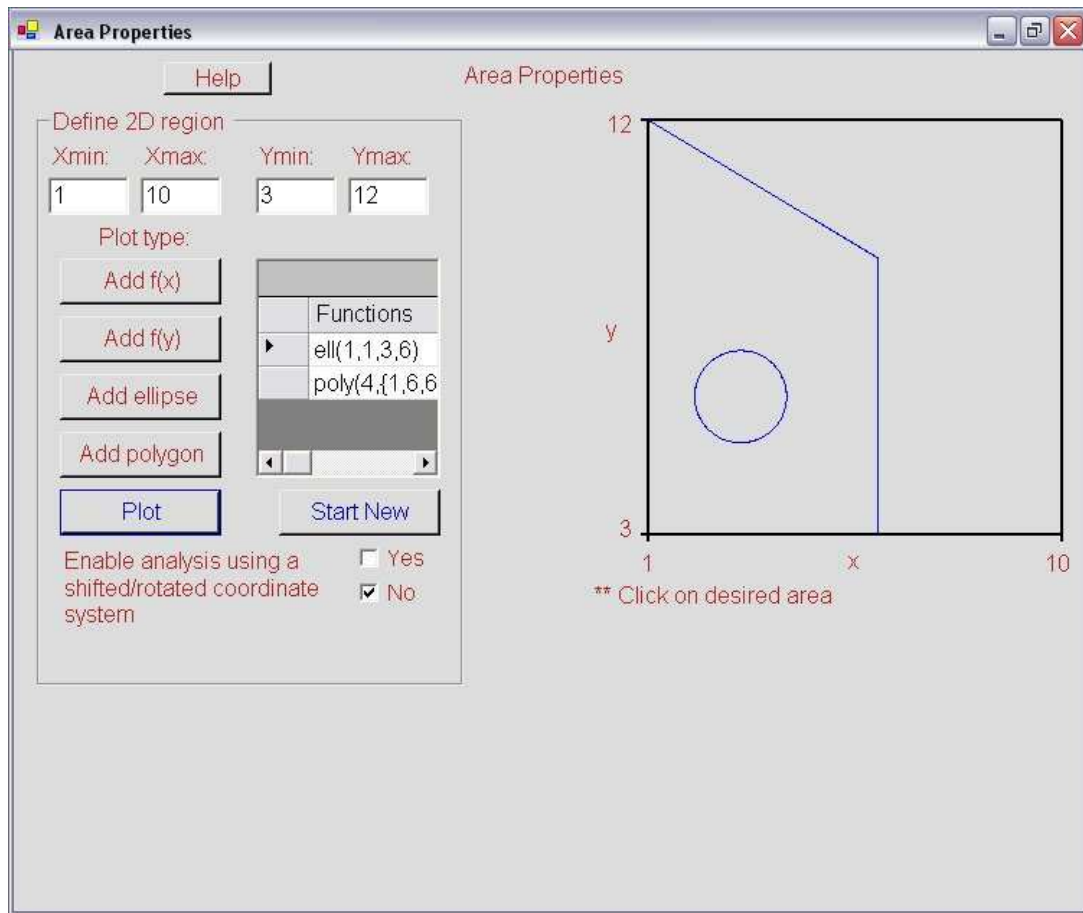


Figure-6: Area diagram.

Click “Yes” if you wanted to use the parallel/rotation axis theorems.

Click anywhere in the desired area to highlight area (you can deselect highlighted areas and you can highlight several areas by several clicks) – you will get the window of Figure-7.

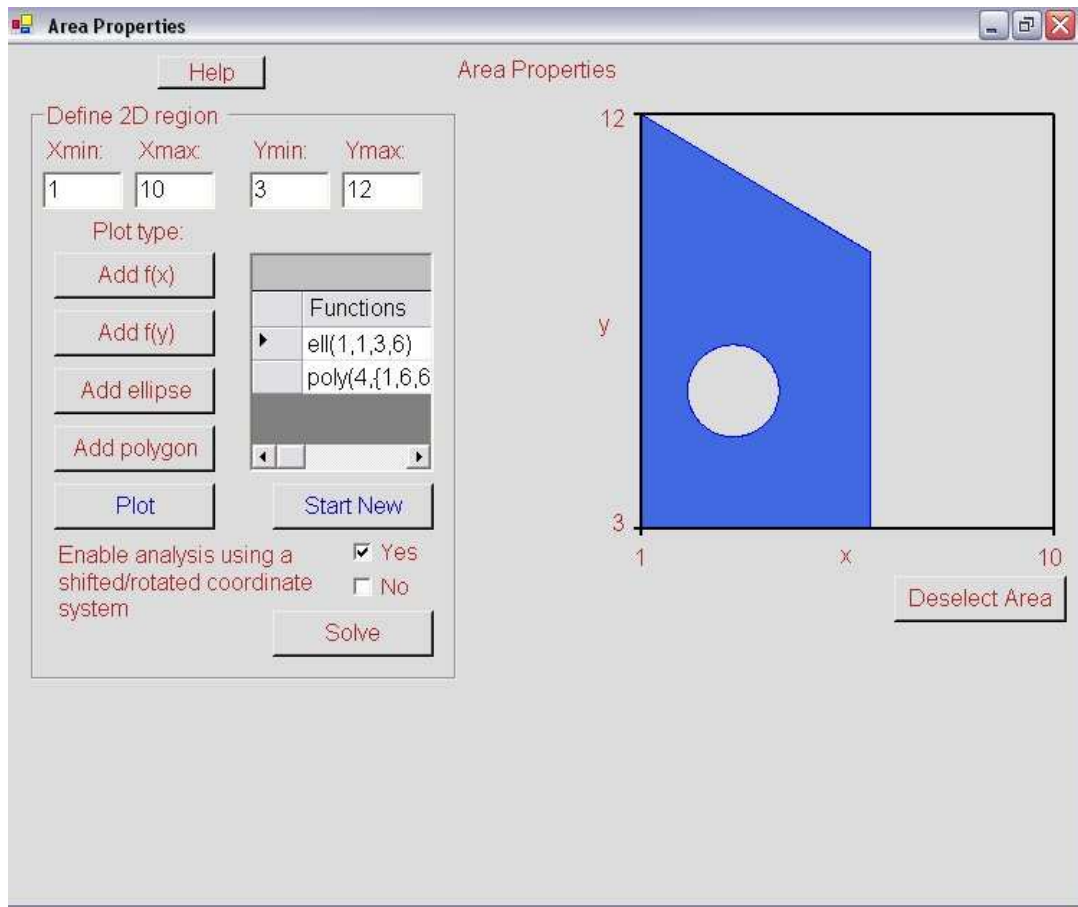


Figure-7: Calculation window.

Click “Solve” – You will get the window of Figure-8.

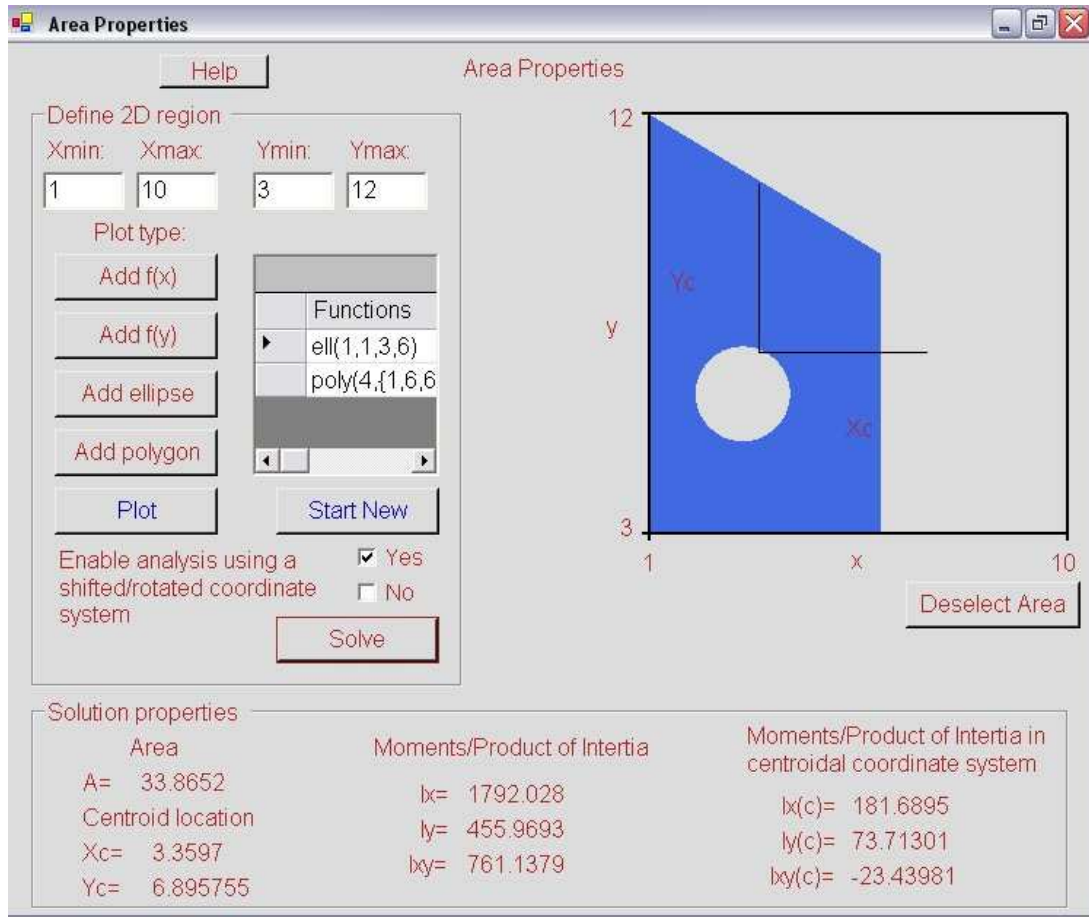


Figure-8: Area Properties

Note: The diagram shows the location of the centroid and the centroidal axes x_c , y_c . Click “Yes” for “Shift” and “Rotate” in the window of Figure-9. Enter coordinates of shifted origin and rotation angle as shown in Figure-9.

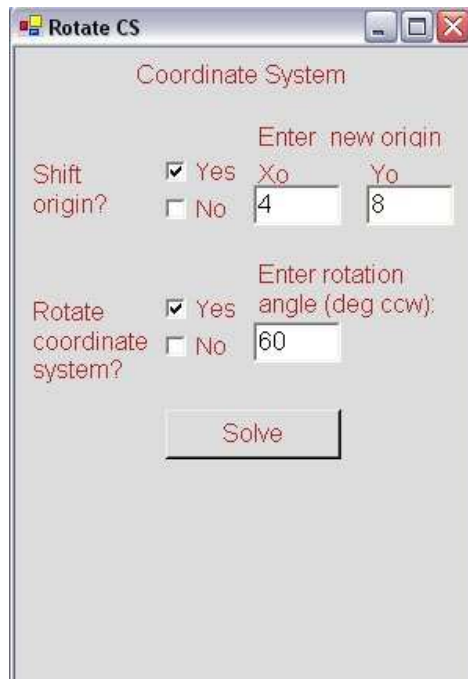
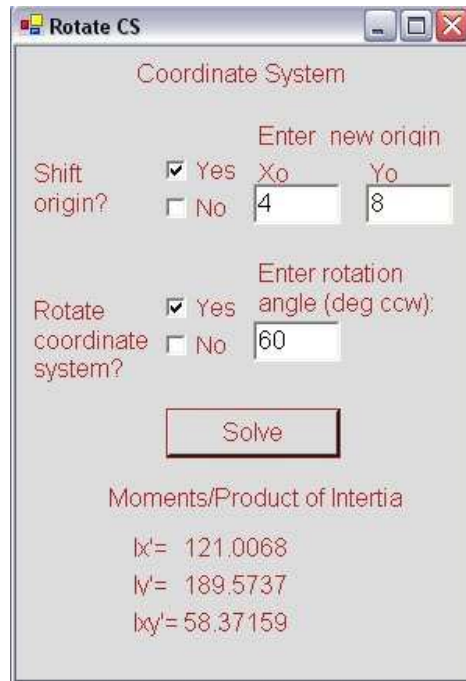


Figure-9: Parallel and rotation axis.

Click “Solve” – You will get the window of Figure-10.



The image shows a software dialog box titled "Rotate CS". It contains the following elements:

- Coordinate System** header.
- Shift origin?** section with a checked "Yes" radio button and input fields for $X_0 = 4$ and $Y_0 = 8$.
- Rotate coordinate system?** section with a checked "Yes" radio button and an input field for a rotation angle of 60 degrees.
- A **Solve** button.
- Moments/Product of Inertia** section displaying the results:
 - $I_{x'} = 121.0068$
 - $I_{y'} = 189.5737$
 - $I_{xy'} = 58.37159$

Figure-10: Transformed inertias.

Example-2

We wish to determine the properties of the area bounded by the x -axis and the curves

$$y = \cos\left(\frac{\pi x}{2}\right)$$

$$x = y^2$$

$$0 \leq x \leq 1, 0 \leq y \leq 1$$

We enter the size of the bounding **square** as shown in Figure-11.

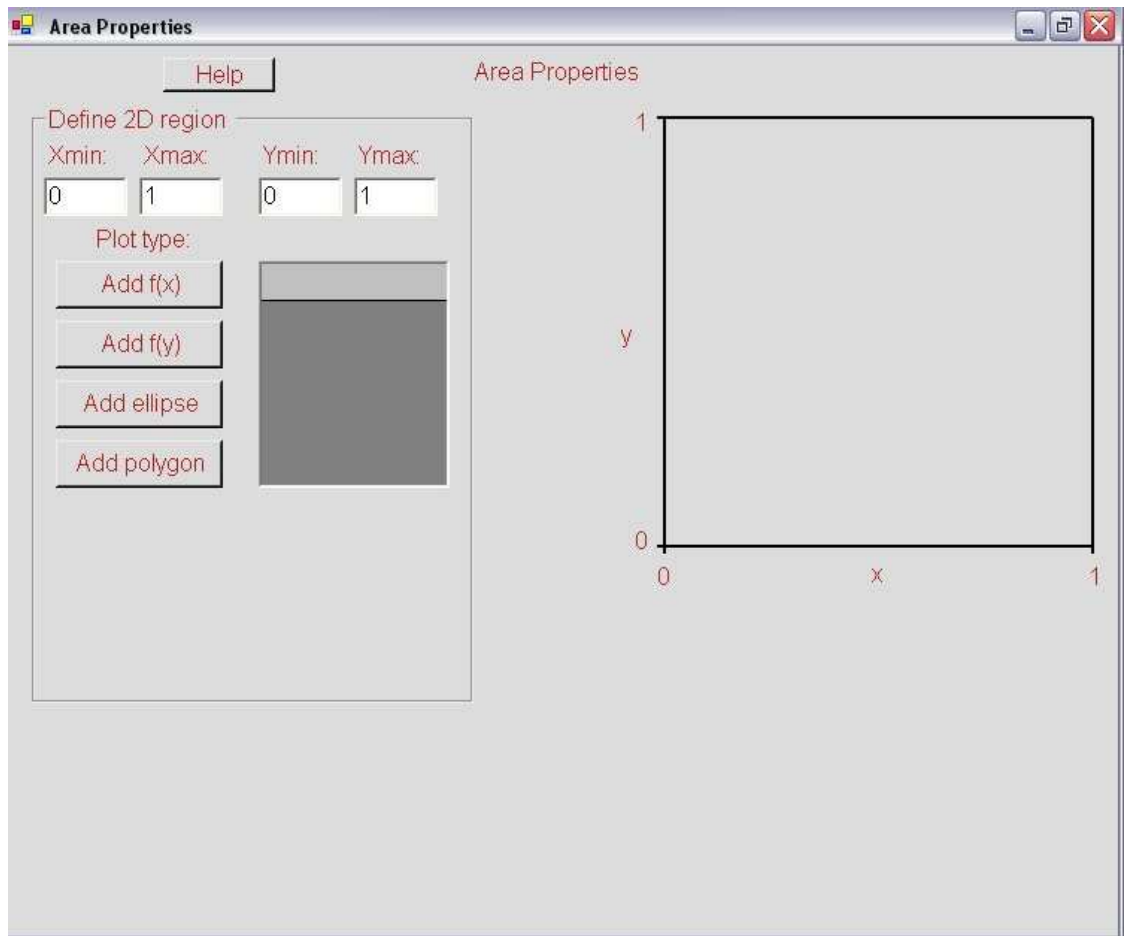


Figure-11 Bounding square, Example-2

Click on “Add f(x)” and enter: $\cos(0.5 \cdot \pi \cdot x)$

Note: π is entered as pi()

Click “Done”

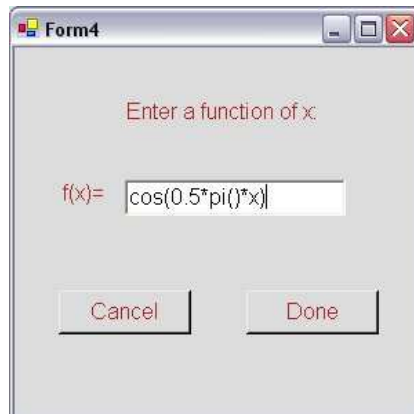


Figure-12: Entering f(x)

Click on “Add f(y)” and enter: y^2
Click “Done”

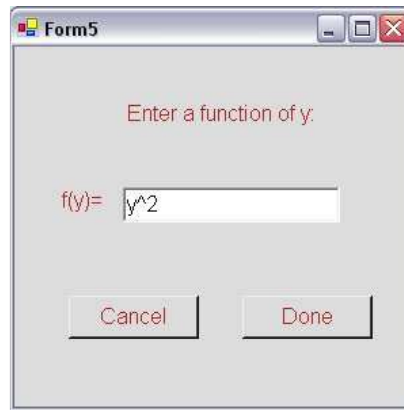


Figure-13: Entering f(y)

Click “Plot” – You will get the window of Figure-14.

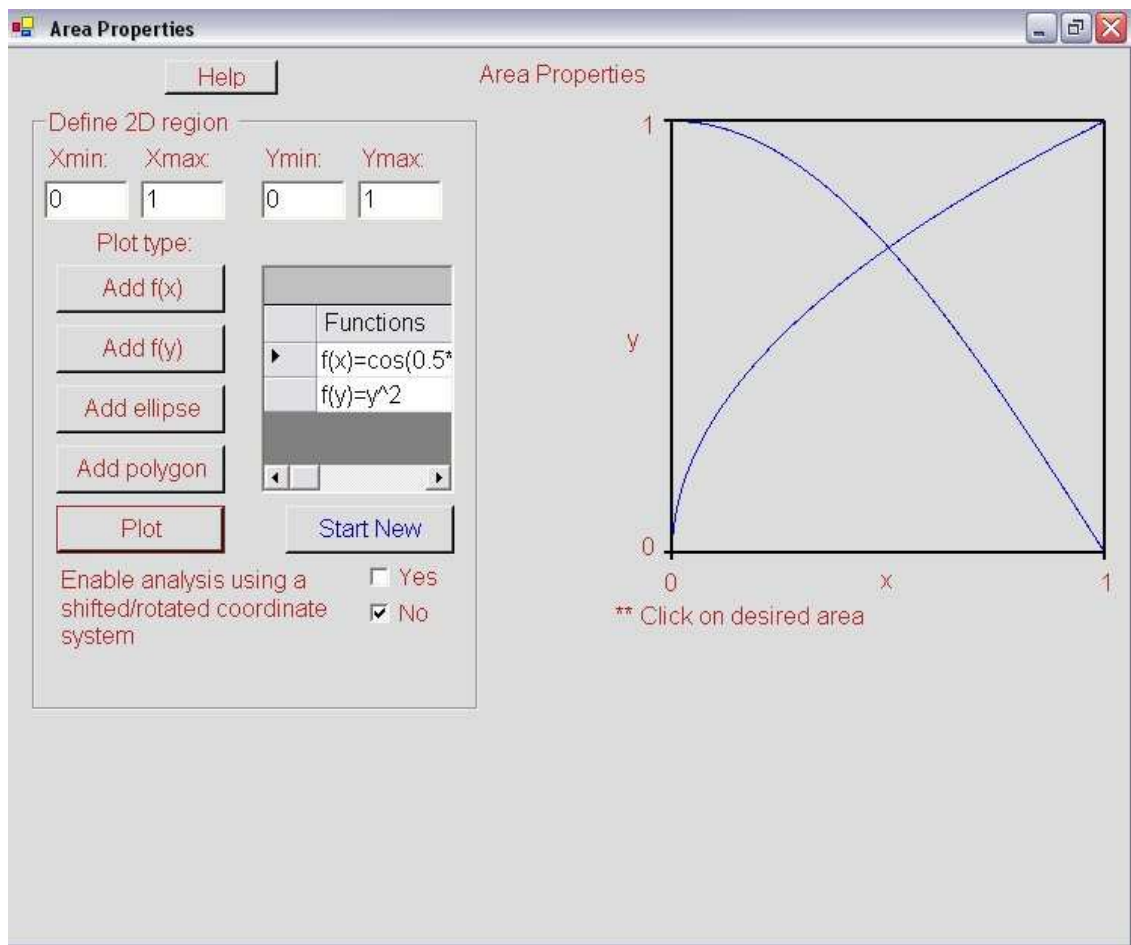


Figure-14: Problem definition, Example-2.

Click “Yes” to enable parallel/rotated axis theorems.
Click on “desired area” to highlight.
Click on “Solve” to get window of Figure-15.

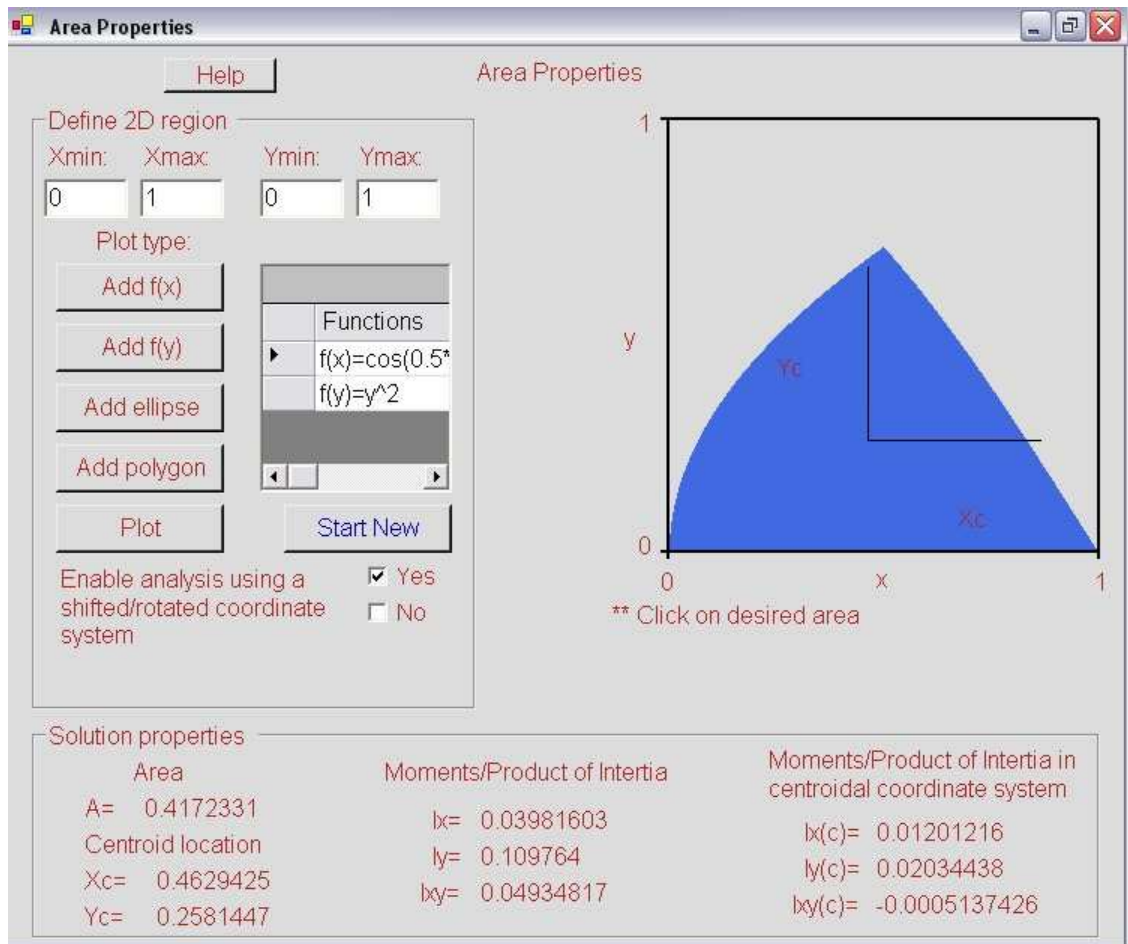


Figure-15: Area properties, Example-2

Use Parallel/Rotated axis theorems on window of Figure-16.



Figure-16: Parallel/Rotated axis theorems, Example-2